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A NEW APPROACH TO THE SOLUTION OF QUASI - UNSTEADY STATE FLOWS USING RIEMANN VARIABLES

By

Halsey B. Chenoweth

and

John J. Fox

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ABSTRACT

This paper presents the generalized non-linear partial differential flow equations of momentum and continuity. These equations are then solved by transformation into the Lagrange form by utilizing a Riemann Invariant.

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LIST OF SYMBOLS

a = $g(p,x)$ = Speed of sound as a function of linear distance and static pressure

A = $F(x)$ = Area of duct as a function of linear distance

k = Adiabatic Constant

P = $u+\ln p$ = Riemannian Invariant for disturbance traveling downstream

p = $e(x,t)$ = Static pressure

Q = $u-\ln p$ = Riemannian Invariant for disturbance traveling upstream

t = Time

u = $h(x,t)$ = Downstream velocity of gas

x = Linear distance

γ = C_p/C_v ratio of specific heats

ρ = Density

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SUMMARY

The solution of the non-linear partial differential equations of momentum and continuity obeying the adiabatic equation of state is presented employing a Riemannian Invariant. The initial boundary value conditions and assumptions are discussed and utilized.

SECTION I. INTRODUCTION

The solution presented in this paper is applicable to the analysis of quasi-unsteady flow in a subsonic diffuser. Assuming a constant duct flow downstream, neglecting heat addition, a method is derived for the analysis of the quasi-unsteady state one dimensional motion of the gas. Methods are discussed for the analysis of non-steady state disturbances and their propagations upstream and downstream. The authors wish to express their appreciation to The Marquardt Corporation, Van Nuys, California, for releasing this paper for publication.

SECTION II. DISCUSSION

The duct air flows are delineated by a series of non-linear partial differential equations based upon the well known laws of conservation of momentum and the continuity equation. The approach to the solution of these non-linear partial differential equations is to linearize them, utilizing the transformation known as the Riemann Invariant.

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Sneddon (ref. 1) and Kantrowitz (ref. 2) among others have treated similar problems employing a different Riemann Invariant.

The Riemann Invariant is a useful tool because it allows the transformation of non-linear partial differential equations, of this type, to yield to the Lagrange solution.

The analysis hinges upon combining the basic equations into a form which is susceptible to algebraic manipulation such that the Riemann Invariant containing the pertinent flow variables is readily recognizable in terms of the Lagrange equation. The result of this manipulation will readily submit to integration providing the characteristic flow variables are known in terms of the independent variables.

SECTION III. ANALYSIS

Assuming a general knowledge of the characteristic flow variables in terms of the independent variables, we may proceed with the following derivation:

The continuity equation is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u A}{\partial x} = 0 \tag{1}$$

and the momentum equation is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \tag{2}$$

for a gas obeying the adiabatic law:

$$p = k\,\rho^{\gamma} \tag{3}$$

and the local speed of sound:

$$a^2 = \frac{dp}{d\rho} = \gamma k\,\rho^{\gamma-1} \tag{4}$$

by transforming:

$$\frac{\partial \rho}{\partial t} = \frac{\rho}{\gamma}\,\frac{\partial \ln p}{\partial t}$$

and:

$$\frac{\partial \rho}{\partial x} = \frac{\rho}{\gamma}\,\frac{\partial \ln p}{\partial x}$$

If equation (1) is expanded, then transformed as above, and cleared of fractions we obtain the following result:

$$\frac{\partial \ln p}{\partial t} + uA\,\frac{\partial \ln p}{\partial x} + \gamma A\,\frac{\partial u}{\partial x} + \gamma u\,\frac{dA}{dx} = 0 \qquad (5)$$

and utilizing equations (3) and (4) we obtain from (2)

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + \frac{a^2}{\gamma}\,\frac{\partial \ln p}{\partial x} = 0 \qquad (6)$$

Subtracting equation (5) from equation (6) results in:

$$\frac{\partial u}{\partial t} - \frac{\partial \ln p}{\partial t} + u\,\frac{\partial u}{\partial x} - uA\,\frac{\partial \ln p}{\partial x} - \left[\gamma A\,\frac{\partial u}{\partial x} - \frac{a^2}{\gamma}\,\frac{\partial \ln p}{\partial x}\right] = \gamma u\,\frac{dA}{dx} \qquad (7)$$

Equation (7) becomes upon combination of coefficients:

$$\frac{\partial Q}{\partial t} + \left\{\frac{\left(A + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}\right)\left(\dfrac{1}{A} - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}\right)}{\left(A + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}\right)\left(\dfrac{1}{A} - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}\right) - 1}\right\}\left\{\frac{A - 1 + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}}{A + 1 + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}} + \frac{\dfrac{1}{A} - 1 - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}}{\dfrac{1}{A} + 1 - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}}\right\} X$$

$$\left(u - \gamma A + uA - \frac{a^2}{\gamma}\right)\frac{\partial Q}{\partial x} = \gamma u\,\frac{dA}{dx}$$

where:

$$Q = u - \ln p \qquad (8)$$

then (8) is of this type:

$$\frac{\partial Q}{\partial t} + f(u,\ A,\ a)\ \frac{\partial Q}{\partial x} = \gamma u\,\frac{dA}{dx} \qquad (9)$$

which is of the form:

$$R\,\frac{\partial z}{\partial t} + S\,\frac{\partial z}{\partial x} = W$$

and is a Lagrange Equation and the method of solution is:

$$dz = \frac{W}{R}\,dt + \frac{W}{S}\,dx$$

so (9) becomes: $dQ = \gamma u \; \frac{dA}{dx} \, dt \; +$

$$\frac{\gamma u}{\left(u - \gamma A + uA - \frac{a^2}{\gamma}\right)} \; \frac{dA}{dx} \left\{ \frac{\left(A + \dfrac{\gamma A^2 - \frac{\gamma a^2}{\gamma}}{u - \gamma A}\right)\left(\dfrac{1}{A} - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}\right)}{\left(A + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}\right)\left(\dfrac{1}{A} - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}\right) - 1} \right\} \left\{ \frac{A - 1 + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}}{A + 1 + \dfrac{\gamma A^2 - \frac{a^2}{\gamma}}{u - \gamma A}} + \frac{\dfrac{1}{A} - 1 - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}}{\dfrac{1}{A} + 1 - \dfrac{1 - \frac{a^2}{\gamma^2 A^2}}{uA - \frac{a^2}{\gamma}}} \right\} dx \qquad (10)$$

This equation has a solution dependent upon the flow and duct variables for a particular duct configuration. Equation (10) generates a family of curves in the xt plane similar to the Method of Characteristics (Landau, ref 3).

For the non-steady duct flow traveling downstream we create a new Riemann Invariant by adding equations (5) and (6) such that the invariant formed is $P = u + \ln p$.

SECTION IV. CONCLUSION

The solution for a specific case of equation (10) renders a linear solution which can be utilized in aerodynamic analysis. The problem of heat addition and non-adiabatic flows becomes extremely complicated. A supersonic or hypersonic flow environment has failed, as yet, to render a solution in closed form.

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APPROVAL

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